Algebra Key Concepts

ACCUPLACER Advanced Algebra and Functions

1. Factoring with the area model

1. Put the first term (ax^2) in the top-left box and the last term (c) in the bottom-right box.
2. Find two numbers that **multiply to a × c** and **add to b**.
3. Write them as x-terms in the two empty boxes.
4. Factor out what each row and each column share. The labels outside the box are the answer.

Example: $2x^2 + 7x + 3$. Here a × c = 6, and 6 + 1 = 7, so the empty boxes get 6x and x.

Tip: if c is negative, the two numbers have opposite signs.

	x	3
2x	$2x^2$	6x
1	x	3

= (2x + 1)(x + 3)

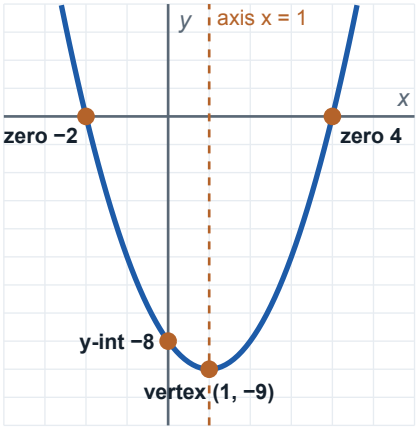
2. Quadratics: three forms of the same parabola

Each form makes a different feature easy to see. All three rows below describe the graph on the right.

FORM	LOOKS LIKE	EASY TO SEE	EXAMPLE
Standard	$y = ax^2 + bx + c$	y-intercept is c	$y = x^2 - 2x - 8$
Vertex	$y = a(x - h)^2 + k$	vertex is (h, k)	$y = (x - 1)^2 - 9$
Factored	$y = a(x - r)(x - s)$	zeros are r and s	$y = (x + 2)(x - 4)$

$$x = \left(-b \pm \sqrt{(b^2 - 4ac)} \right) / 2a$$

quadratic formula: works on any $ax^2 + bx + c = 0$



- If a is positive, it opens up. If a is negative, it opens down.
- Roots, zeros, solutions, and x-intercepts all mean the points where y = 0.
- $b^2 - 4ac$ positive: 2 solutions. Zero: 1. Negative: none.
- Watch the sign in vertex form: $(x - 1)^2$ means h = 1, not −1.

3. Exponents and logs

A log is an exponent: $\log_b x = y$ means $b^y = x$

Example: $\log_2 8 = 3$, because $2^3 = 8$

RULE	EXAMPLE
Multiply: add exponents	$x^3 \cdot x^4 = x^7$
Divide: subtract exponents	$x^7 / x^2 = x^5$
Power of a power: multiply	$(x^2)^3 = x^6$
Zero exponent	$5^0 = 1$
Negative exponent	$2^{-3} = 1/8$

Growth and decay: $y = a \cdot b^x$

- a is the starting amount, b is the factor.
- Grows 5% a year: b = 1.05. Shrinks 5%: b = 0.95.
- Doubles every 4 hours, x in days: 2^{6x} (6 doublings a day).

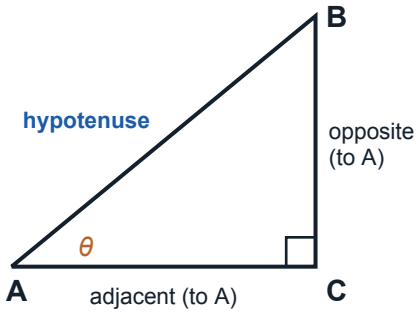
Solving

- Same base? Set the exponents equal. $4^x = 8^{x-1}$ becomes $2^{2x} = 2^{3x-3}$, so $2x = 3x - 3$ and $x = 3$.
- Different bases? Use a log. $2^{5x} = 7$ becomes $5x = \log_2 7$, so $x = (\log_2 7) / 5$.

4. Trig with right triangles

SOH: sin = opp / hyp	CAH: cos = adj / hyp	TOA: tan = opp / adj

- The hypotenuse is across from the right angle. It's always the longest side.
- $a^2 + b^2 = c^2$. Common triples: 3-4-5 and 5-12-13.
- The two small angles add to 90°, so sin A = cos B.
- Degrees to radians: multiply by π/180. So 180° = π.



ANGLE	SIN	COS	TAN	SIDES OF THE TRIANGLE
30°	1/2	√3 / 2	√3 / 3	30-60-90: 1 : √3 : 2
45°	√2 / 2	√2 / 2	1	45-45-90: 1 : 1 : √2
60°	√3 / 2	1/2	√3	